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Credit Risk of Financial Vehicle for Energy Retrofits of Buildings V01

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Content

1 Overview

If we assume that the observed historical rates of energy retrofits of buildings (energetic renovation of building envelopes and heating replacement with renewable energy source) is maintained in the future, significant part of the existing real estates will not meet the climate goals of Switzerland by 2050. In spite of undisputable benefits of sustainable investments for the environment and the economy (e.g. mitigation of price risk and energy shortage), there seem to exist obstacles which are slowing down the energy retrofits of buildings. The following flow-diagram displaying the expectations of the involved stakeholders/players can give an insight into obstacles from financing point of view.

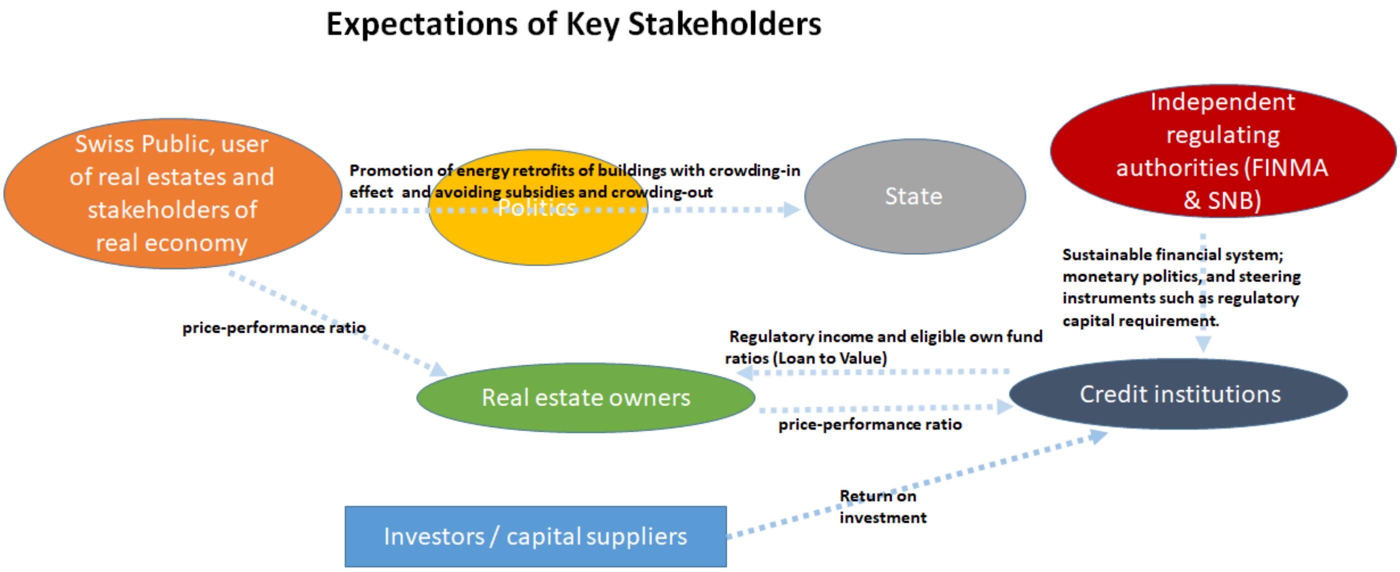


Figure 1: Expectation flow-diagram

Our intention with the above flow-diagram is to detect the financial gap if there is any. Therefore, it focuses on the expectations within financial system. It displays the expectations triggering the real economic activities, but excludes the further details of value chain, economic interactions, demand/ supply relation, etc. within real economic frame, which are rather subject to other RENOWAVE work streams. The direction of an arrow together with its associated text indicates the expectation of a stakeholder from its neighbor stakeholder.

Although independent, with different range of actions, the regulation organs FINMA and SNB have common goal. Namely, to ensure sustainable financial system on a broad scale. From these two organs, let us draw our attention on FINMA which directly concerns us. Among others, the mandate of the FINMA is the protection of insured persons, small savers, small investors, etc. that might be exposed to risky operations of insurers and banks. The potential instabilities of financial system experienced in the history (e.g. cause of the financial crisis in 2008) urged the FINMA to regulate these institutions more and more, with positive, but also undesired side effects for some stakeholders. In our case, it is the mortgage market that plays role. To avoid mortgage, and with it, real estate crises FINMA regulates mortgage market by setting thresholds on *income/expense ("Tragbarkeit")* and *loan to value ("Belehnung")* (LtV)[1] ratios of the real estate owners.

The regulatory capital (the amount of capital a bank or other financial institution has to have as required by its financial regulator) depends on the risk profile of the real estate owners which is among others defined by these ratios and relevant for the pricing of the mortgage rate (because of cost of capital as a pricing component). High regulatory capital requirements can therefore increase the opportunity cost of the loans for the lending institution, leading to high mortgage rates. The low rates at which energy retrofits of buildings are implemented, suggests, for a significant part of the real estate owners, the financial conditions don't meet their expectations to undertake energy retrofits. They either lack sufficient eligible own funds or/and they incur economical extra costs which they cannot fully pass to tenants if the object is rented, for which one among several explanatory factors can be the mortgage rate. These arguments support the hypothesis that there is a financial gap which is not necessarily the only driver of the low energetic real estate retrofit rates, but its removal seem to be a necessity to increase the rate. The ensuing question, which we address next, is which financial vehicle removes this gap, in a way that could meet the expectations of stakeholders, and respect existing FINMA requirements.

Removal of the Financial Gap

Any financial vehicle which is supposed to remove the financial gap effectively, has to comply with the expectation of the key stakeholders in Figure 1. From mortgage market and to some extend political point of view this means:

- *Promoting crowding-in and keep crowding-out at an acceptable level*
- *to avoid subsidies as much as possible*
- *mortgage loans must satisfy the economic expectations of the real estate owners and tenants. Especially, the transfer of amortization (partially or full) of ecologic investments to the tenants shouldn't cause inacceptable raise of rents.*

[1] For more details we refer to:
https://www.swissbanking.ch/_Resources/Persistent/0/e/3/f/0e3fe72b0bdc557fef84893287ece62b37172e4c/SBVg_Richtlinien_betreffend_Mindestanforderungen_bei_Hypothekarfinanzierungen_DE.pdf

An approach which satisfies these conditions may have an increased chance to convince the real economy market players and avoid obstacles. It might therefore be more effective than alternatives.

The first condition is satisfied if our approach is limited to a subset of owners that are, because of mentioned regulating ratios, out of current scope of credit institution. The second and third conditions are closely related and rule out many traditional forms of support. They can, however, be met by a risk sharing scheme for supplementary credit for energetic retrofits of buildings which we would like to illustrate with the following diagram.

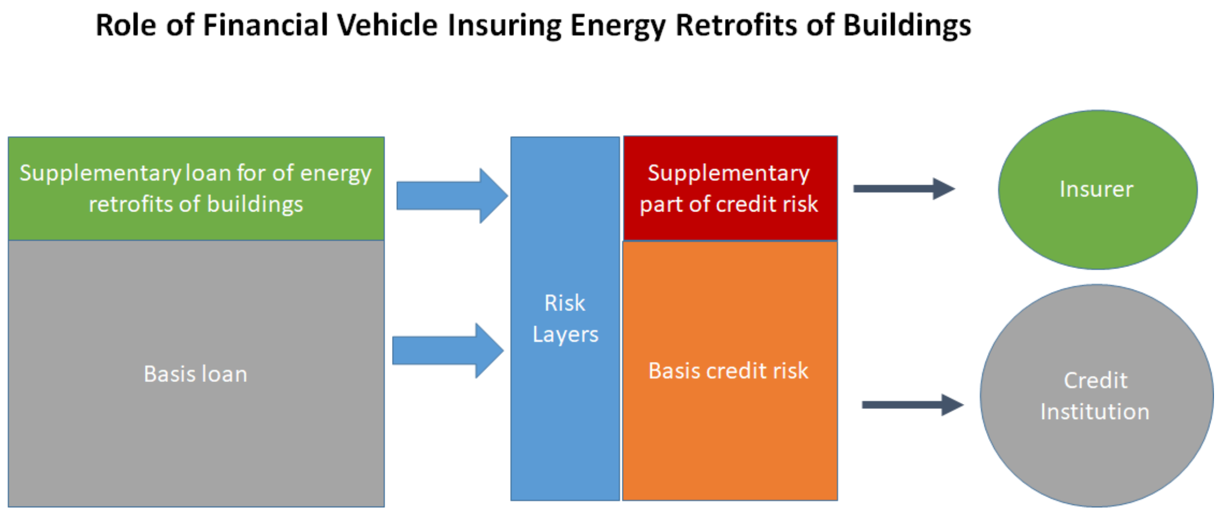


Figure 2: Insurance as financial vehicle

As mentioned before FINMA, the independent supervisory authority over Swiss financial institutions, regulates, the regulatory capital required for these institutions, but with different capital requirements regimes for the banks (based on Basel III) and insurers (Swiss Solvency Test, SST) for good reasons. In our case, these differences may address to the bottleneck in channeling capital flows in direction of energy retrofits of buildings. In the above diagram, the excess of regulating thresholds (because of supplementary credit for ecological renovation) of the mentioned ratios need well-reasoned exception to policy and/or higher regulatory capital.

In this sense, we suggest the decomposition of total credit risk exposure in *basis loan* and *supplementary loan* that could allow credit institutions, via a financial vehicle, acting as an insurer, to optimize the regulatory capital required, and with it, the capital costs. In other words, the credit institution increases the loan, from the *basis* loan size to *basis + supplementary* loan, but the portion added is treated separately, and subject to the shared special mortgage insurance scheme we design. The credit institution pays a premium for the insurance of that supplementary loan part, covering the cost of mitigating of the additional risk generated by the supplementary credit, but in return the institution gets relief of required capital.

The introduced risk sharing scheme aims to address the concerns of independent authorities such as FINMA and SNB with respect to stable financial system and real economy respectively. Firstly, with this approach, system relevant credit risk event on large scale can be mitigated at least to some extent. Secondly, this approach incentivizes the substitution of non-renewable energy carriers with renewable energy carriers. This substitution is not only from ecological, but also from real economic point of view important. The recent global crises prove that the emerging energy supply and price risk could be mollified significantly by more diversification through more intensive ecological substitution, which we think can be accelerated by the suggested approach.

Scope of this paper

Cost of capital represents an important component of the mortgage rate. The impact of risk sharing approach depends on the benefits of credit institutions and the price of shared risk. The aim of this paper is to prove that the required capital of an insuring vehicle is low enough to allow acceptable insurance premium that might accelerate noteworthy market demand for sustainable renovations/investments of real estates. For this purpose, we will define model points representing risk classes and measure the required capital based on major risk metrics and compare them with the existing standard regulations in absence of the suggested regulation. Although the following issues are partially relevant for our purpose, their precision will be out of scope in this paper:

- *Capital structure of the institution*
- *Actuarial pricing parameters (expenses, insurance risk, cost of capital)*
- *Juristic form*
- *Structure of the institution and governance*
- *Non-residential real estates*

Instead, we will make assumptions on the safe side where it is needed.

Main Result

Main result of this paper is that the required capital for sound financial vehicle as insurer leaves enough marketable product from pricing point of view that can accelerate the ecological renovations. Which can be summarized as follows: By defining a threshold below the official income ratio (reflecting the risk appetite of capital suppliers) and LtV of 80% of the real estate owners, we can define a potential target group of residential real estate owners that are currently due to high cost of capital "not eligible" for a supplementary credit (see „Richtlinie betreffend Mindestanforderungen bei Hypothekarfinanzierung" der Schweizerischen Bankiervereinigung). This target group can be decomposed in risk classes with certain default probabilities (PD) paired with objects classes backing the loans (specified by loss given default which is abbreviated by LGD). It turns out that the suggested financial vehicle acting as an insurer needs required capital of less than 1.55% of insured exposure which is significantly lower than even eligible mortgage credits under standard requirements.

This shows that a financial vehicle acting as an insurer would allow the credit institution to offer a sustainable mortgage product for their new business that is profitable and attractive for real estate owners.

The required capital of 1.55% of exposure is on the safe side, when the real estate/mortgage market shock plays an important role. This persistent shock lasted ca. six years and can be seen as the "fallback" of expansive monetary politics SNB from the eighties to stimulate the economy and/or to mitigate the market shock in 1987 in the US together with other events favoring huge money supply additionally. For example, almost at the same time as the electronic payment system "Swiss Interbank Clearing (SIC)" was introduced. Moreover, the mortgage market was by far not rigorously regulated comparing to current regulation. The natural follow-up of accumulation of these events was acceleration of money circulation and higher "market temperature" favoring a climbing inflation, whichis the main the responsibility of SNB. This increasing inflation urged the SNB to reduce the monetary volume by increasing the interest rates followed by additional chain reactions in the nineties. In this period, the systematic drivers of credit risk mortgages were persistently unfavorable: high interest rates, high unemployment and high inflation[2], with a consequence of 1.4% annual write-off in average of credit institutions for approximately six years.

The relevant credit risk components are default probability (PD), loss given default from insurer's point of view (LGD), Exposure at Default (EaD), and the systematic parameter q controlling the non-diversifiable risk due to systematic drivers, which is a crucial driver of the required capital. For risk modelling purposes, the crises of the nineties are incorporated into our estimations in two ways. Firstly, for the best estimate of q is 63%. The observed top-down write-offs in the crisis of the nineties was used in order to determine this parameter (see Appendix). Secondly, risk models reflecting the "normal world" don't have enough heavy tail to cover the historical shocks with an expected frequency and are therefore not satisfactory from a risk management point of view. Especially in our case, where we are dealing with credit risk, this model feature plays an even more crucial role. Therefore, this shock was included in the risk model by means of an SST methodology.

Following table shows the sensitivities of required capitals of both capital requirement regimes with respect to control parameter q. It also shows how important it is to incorporate observed real estate shocks of nineties in Switzerland.

q	Before schock		After schock	
	Required capital for A (before schock)	Required capital for SST (before schock)	Required capital for A (after schock)	Required capital for SST (after schock)
0.6	0.493%	0.242%	1.508%	1.428%
0.65	0.624%	0.291%	1.515%	1.429%
0.7	0.783%	0.349%	1.530%	1.430%

Table 1: Sensitivities of required capital of financial vehicle with respect to systematic parameter q

[2] Refer to https://snbchf.com/wp-content/uploads/2014/01/Tatsiana-Meier-Real-Estate-Crisis.pdf for details.

From table 1 we first conclude that the required capital for an A rating before incorporating the shock of the nineties clearly dominates the required capital according to SST. But, after the shock, this dominance becomes much milder. Luckily, in a neighborhood of estimated q, we don't observe high sensitivities of both required capitals. However, the full table and graph in section 6 show that the slope start to increase and becomes critical as q becomes large.

Please also note that along our parametrization and modelling process, we follow the general actuarial "safe side" rule; for each step where we encounter parameter or model uncertainty due to lack of granularity of observed data or model risk, we prefer conservative approach.

How to read this paper

In section 2 we will specify the modelling simplification and introduce the risk metrics which are relevant from the point of view of FINMA. The section 3 is devoted to definition of the insurance product, followed by section 4 for modeling of loss distribution of the Insurer in which the real estate shock of nineties also incorporated. In section 5 we will introduce the assumption made. Finally in section 6 we will present the results for required capital and compare them with the existing standard required capital in absence of the financial vehicle.

2 Simplifying Assumptions and Capital Requirement Metrics

Globally, there are several required capital regimes based on a different metrics. In spite of similarities to some extend between them, there are also crucial conceptional differences implying significant quantitative deviances.

In Switzerland, regulatory organs for private insurance and foundation acting as insurer are different and have different capital requirements. In the first case, the FINMA is the relevant regulator, whereas the latter is regulated by the regulation organs of foundations ("Stiftungsaufsicht"). However, since most credit institutions are regulated by the FINMA, even in the latter case, the

FINMA will play a crucial role. Because relief of required capital depends on the rating of the insurer by the FINMA-recognized rating agencies. In other words, credit institutions regulated by the FINMA, but without an internal model must comply with the standard Basel rules implying a rating quality (for capital relief purposes) of insurer even if the insurer is not supervised by the FINMA (in case of insurers juristic form is foundation). Hence, on the safe side, we will estimate the required capital according to following metrics:

- Required capital according to Swiss Solvency Test (SST)
- Required capital targeting a rating that allows capital relief

Before we define the associated metrics of above regimes, we will make the following **simplifying assumptions:**

- Relatively low capital need (which will be evident) in conjunction with favorable capital structure (e.g. by means of catalytic involvement of the state or non-profit organizations), the relatively low expected insurance claims and the flexible pricing instruments (e.g. bonus-malus), allow enough pricing negation room, so that the expected insurance income due to insurance premiums cover all the yearly cashflow-out consisting of expected insurance claims, expenses and capital costs (return on capital).
- Risk free assets (cash) are the only assets, backing the liabilities and generating zero income.

The first assumption implies that the break-even point is already reached, that is, the financial vehicle has acquired a loan portfolio of sufficient mass and does not need any sort of startup funding anymore. It also implies enough room so that especially the *Market Value Margin* (MVM) is hidden in the future premiums agreed with the insured credit institutions, which ensures that in case of a run-off state of the insurer, a rational investor will be willing to take over the insured portfolio (contractually defined future premiums and claims generated by the portfolio as well as the future expenses). With the second assumption we use a fallback option for the asset backing the liabilities. This fallback option is truly available to the insurer and for our work it has the advantage that it leads to no asset-related uncertainties that would otherwise have to be modelled.

If we denote the insured risks claims at horizon (t=1) with $(\mathrm{Loss_i})_{i\leq N}$, the above simplifying assumptions imply that $\mathrm{RBC_1} = \mathrm{RBC_0} - \sum_{i\leq N} \mathrm{Loss_i}$. Where $\mathrm{RBC_t}$ is the risk bearing capital at year t ("risikotragendes Kapital" $\mathrm{RTK_t}$ in the Swiss Solvency Test SST). It follows that, under the mentioned simplifications we have

(1) $\Delta\mathrm{RBC} = \sum_{i\leq N} \mathrm{Loss_i}.$

Details are found in https://www.finma.ch/FinmaArchiv/bpv/download/d/SST_technischesDokument_061002.pdf.

In this case, minimal capital requirement according to SST is given by

(2) $\mathrm{RBC_0} \geq \mathrm{ES_{1\%}}[\sum_{i\leq N} \mathrm{Loss_i}]$

The above requirement is equivalent to saying that $\mathrm{RBC_0}$ must be at least as high as the average of the 1% worst business scenarios (measured according to market consistent valuation principles). In other words, RBC_0 should be larger than the Expected Short Fall (ES) with respect to the 1%-quantile[3].

Second requirement is defined by the default probability of the institutions and imposes that the probability of default of the institution should not be higher than the probability of default of an institution with A:

(3) $P\,[\mathrm{RBC} - \sum_{i \leq N} \mathrm{Loss}_i \leq 0] \leq P[\text{Default of insurer with A rating}]$

[3] For more insight into Expected Shortfall (ES) we refer to following articles:
https://www.sciencedirect.com/science/article/abs/pii/S0378426602002832
https://arxiv.org/pdf/cond-mat/0105191.pdf
https://www.finma.ch/FinmaArchiv/bpv/download/e/SST_techDok_061002_E_wo_Li_20070118.pdf

3 Contractual Claims of the Insurance Product

As indicated in the introduction, the suggested product should insure exactly the losses in case of default which can be clearly assigned to supplementary credit of real estate owner i. For this purpose, we first define the *joint losses* (losses that should be shared by credit institution and the insurer) in case of default of i. Formally:

(4) $G_i = \max\left(B_i + Z_{B_i} + M_i + Z_{M_i} - W_i, 0\right)$

where,

B_i = Basis credit of i

Z_{B_i} = Losses due to write off of mortgage income on basis credit

M_i = Supplementary credit of i

Z_{M_i} = Losses due to write off of mortgage income on supplementary credit

W_i = Selling price of the real estate owned by i

The above definition of G is straight forward and says that the joint loses of the credit institution and insurer together consist of basis credit, supplementary credit, mortgage rates on these exposures after deducting the income due to liquidation of the real estate if this value is positive. In other words, if the revenues obtained from liquidations of real estate exceed the $B + Z_B + M + Z_M$, then the join loss is zero.

Now we can define the claim of insured credit institution in case of default of real estate owner as follows:

(5) $L_i = \min\left(G_i, M_i + Z_{M_i}\right)$

The formula (5) defines at the same time the loss which is assigned to insurer which takes place if and only if $G_i \geq 0$ and can be at most $M_i + Z_{M_i}$.

By plugging (4) into (5) we obtain:

(6) $L_i = \min\left(\max\left(B_i + Z_{B_i} + M_i + Z_{M_i} - W_i, 0\right), M_i + Z_{M_i}\right)$

The above formula says that the insurer takes over part of the joint losses in case of the default of the real estate owner i, that is induced by the supplementary credit. Note also that the random variable loss L_i is known when the case is closed and depends on the liquidation date which is itself a random variable.

4 Modeling of Loss Distribution of the Insurer

For a given set of real estate owners i, $\mathrm{i}=1,2,\ldots\mathrm{N}$ we are interested in the distribution of the portfolio loss $\mathrm{L}=\sum_{\mathrm{i}\leq\mathrm{N}}\mathrm{Loss_i}$ of the insurer to compute the introduced target capitals in section 2 which we will be the objective of this section.

4.1 Modeling of Credit Risk Drivers

For each owner i the loss $\mathrm{Loss_i}$ of the insurer can be written in modular manner as follows:

(7) $\mathrm{Loss_i}=\mathrm{D_i}*\mathrm{L_i}$

In (7) $\mathrm{D_i}$ takes the value one if the owner i is defaulted and zero in all other cases. $\mathrm{L_i}$ can be interpreted as "conditional random variable" in case of default of the owner i (see by (5) or (6) in section 3). However, in what follows we will omit this stochasticity and write:

(8) $\mathrm{Loss_i}=\mathrm{D_i}*\mathrm{LGD}$

Where LGD is the *expected loss given default* and depends on the real estate class of the owner i. The LGD in this section is the one from insurers point of view and related to the introduced product in section 3. Please also note that this simplification is usual practice and can be compensated via sensitivity analysis and/or a conservative choice of LGD on the safe side. In our case, we will have two classes of residential real estate LGD classes. Namely, own used and rented real estates.

Next we will model the components of $\mathrm{D_i}$ and LGD in (8).

Modeling of $\mathrm{D_i}$

Reasonable risk measures are sensitive with respect to systematic risk drivers, which we will consider in the default event of the owners.

Let $\mathrm{PD_i}$ denote the default probability of i:

(9) $\mathrm{PD}=\mathrm{P}[\mathrm{D_i}=1]$.

$\mathrm{D_i}$ can be modeled as follows:

(10) $\mathrm{D_i}=1_{[\mathrm{R_i}\leq\mathrm{PD}]}$.

Where $\mathrm{R_i}$ is a uniformly distributed random variable in interval [0,1].

Given a continuous invertible distribution function $\mathrm{F_{X_i}}$ of random variable $\mathrm{X_i}(.)$, $\mathrm{D_i}$ can also be modeled as:

(11) $\mathrm{D_i}=1_{[\mathrm{X_i}\leq\mathrm{DT}]}$,

where DT is the default-trigger and defined by $\mathrm{DT}=\mathrm{F}_{\mathrm{X}_i}^{-1}(\mathrm{PD})$.

In order to see this first observe that

$$\mathrm{P}\left[\mathrm{F}_{\mathrm{X_i}}^{-1}(\mathrm{R_i}(.))\leq\alpha\right]=\mathrm{P}\left[\mathrm{R_i}(.)\leq\mathrm{F_{X_i}}(\alpha)\right]=\mathrm{F_{X_i}}(\alpha)=\mathrm{P}[\mathrm{X_i}\leq\alpha]$$

It follows that

$$P[1_{[X_i(.)\leq DT]} = 1] = P[X_i(.) \leq DT] = P\big[F_{X_i}\big(X_i\,(.)\big) \leq F_{X_i}(DT)\big] = P[F_{X_i}(X_i\,(.)) \leq PD]$$

$$= P[F_{X_i}(F_{X_i}^{-1}(R_i(.)) \leq PD)] = P[R_i(.) \leq PD] = PD.$$

Hence, D_i according to (10) and D_i according to (11) have the same distribution.

The general model formula (11) is very useful and allows to incorporate the systematic drivers of default on top of the idiosyncratic ones. For this purpose, let as assume that default of owner i is driven by a $N(0,1)$ random variable X_i, which is the sum of systematic and idiosyncratic random variables:

(12) $X_i = S + e_i$

In (12) random variables S and e_i are assumed to be independent and normal distributed according to $N(0, q^2)$ and $N(0,1 - q^2)$ respectively, so that X_i is $N(0,1)$.

Modelling of LGD

Assume that we observe a default of an owner at horizon (t=1). Strictly speaking, at valuation date (t=0) we must model the value of L_i in a market consistent manner at horizon which can be interpreted as a pay-off function of underlying real estate according to the product defined by (5) or (6) and endowed with “optimal” selling strategy. Moreover, although the insured credit institution can be seen as the holder of the option, it is not completely free in exercising this option. In order to avoid all these technical and formal complications for valuation of this "American like” derivative,[4] as-of horizon we will proceed pragmatically but on the safe side as follows: First step is to determine the annual variances of prices of own used and rented real estates Var_C and Var_R respectively. Moreover, we will assume that the distribution of the normalized prices W_C and W_R are $N(1, Var_C)$ and $N(1, Var_R)$. The normalized prices of W_C and W_R can be modelled by a Brownian motion with initial value 1 and annual variances Var_C and Var_R at time t=1 which are martingale. Although other diffusion processes would be from methodological point of view more coherent (e.g. geometric Brownian motion with zero drift after discounting and ruling out negative values) we feel more comfortable with this rather conservative approach.

By using (6) we obtain LGD_E and LGD_M as follows:

(13) $LGD_C = E\,[\min(\,\max(B + Z_B + \, M + Z_M - W_C, 0), M + Z_M\,))]$

and analogously,

(14) $LGD_R = E\,[\min(\,\max(B + Z_B + \, M + Z_M - W_R, 0), M + Z_M\,))]$.

[4] American” because there’s no fixed exercise date, but only “American-like”, because there’s no fixed expiry date.

Here $E[.]$ in (13) and (14) denotes expected value (which is risk neutral due to martingale property of Brownian motion). A closed formula for the above derivative value is highly challenging (if not impossible). Instead, we used Monte-Carlo simulations in order determine these values.

In what follows, with exposure we will mean the exposure from insurers point of view unless otherwise is stated, which is M. Moreover, LGD_C and LGD_R will be presented in % of the exposures.

We used time series of Wüest & Partner (W&P) for the variances of the underlying real estates (which also includes the period of real estate shock in nineties) and the following conservative assumption:

LtV before supplementary credit=66%

LtV after supplementary credit=80%

Duration of liquidation =1.5 year

Write offs of mortgage rates= 3%

With these assumptions and mentioned Monte-Carlo simulations we obtain less than 1% and less than 2% for LGD_C and LGD_R respectively. However due to sensitivity of required capital with respect these parameters and to have a completely sound results, in this paper, we will use of 2% and 4% for LGD_C and LGD_R respectively.

4.2 Loss Distribution of Insured Portfolio

Assume each real estate owner i with supplementary credit $exposure_i$ is a member of a risk class C specified by a pair $C = (PD, LGD)$. Portfolio loss can be modeled by

(15) $Loss = \sum_C \sum_{i \epsilon C=(PD,LGD)} 1_{X_i \leq DT(PD)} . LGD_i . exposure_i$

Where X_i is defined according to (12) and $DT(PD)$ is the default trigger associated to PD.

The distribution function of Loss will be denoted by F_{normal}.

Incorporation of real estate crises of nineties

The F_{normal} defined as above covers the systematic risk driver S. However, the distribution function F does not necessarily have fat enough tail to cover the extreme events as observed in nineties with an expected frequency. To incorporate the real estate crises like nineties with a frequency λ we put

(16) $F = (1 - \lambda).F_{normal} + \lambda.F_{shocked}$

Here

$F_{shocked}(x) := F_{normal}(x - SoS)$

Where, SoS means the severity of shock.

The F as defined in (16) ensures that a shock of nineties with observed top-down severity of SoS of credit institutions will occur with a frequency of λ.

5 Assumptions

5.1 Portfolio Structure

Following table displays the mid and long run portfolio structure of the insurer

		Own used	Own used	Own used	Own used	Rented	Rented	Rented	Rented	
		1	2	3	4	5	6	7	8	Topdown
Portfolio Structure	Default	0.2%	0.4%	0.2%	0.4%	0.2%	0.4%	0.2%	0.4%	0.3%
	LGD	2.0%	2.0%	2.0%	2.0%	4.0%	4.0%	4.0%	4.0%	2.9%
	Supplementary credit in 1'000 CHF	150	150	150	150	400	400	400	400	
										Total
	Number of loans	1'750	1'750	1'750	1'750	538	538	538	538	9'150
	Exposure in 1'000 CHF	262'500	262'500	262'500	262'500	215'000	215'000	215'000	215'000	1'910'000

Table 2: Mid and long run portfolio structure

In the above table, each pair consisting of *default probability* and *LGD in % of supplementary credit* represents a risk class.

Following table displays the Default Triggers which we used in this paper:

	PD	DT
PD class1	0.002	-2.878
PDclass2	0.004	-2.652
top-down	**0.003**	**-2.748**

Table 3: Default Triggers

5.2 Systematic driver

For systematic parameter q we refer to Appendix

5.3 Shock Parameter

For shock parameter we will use

λ =1/30

SoS =observed annual write offs =1.4%

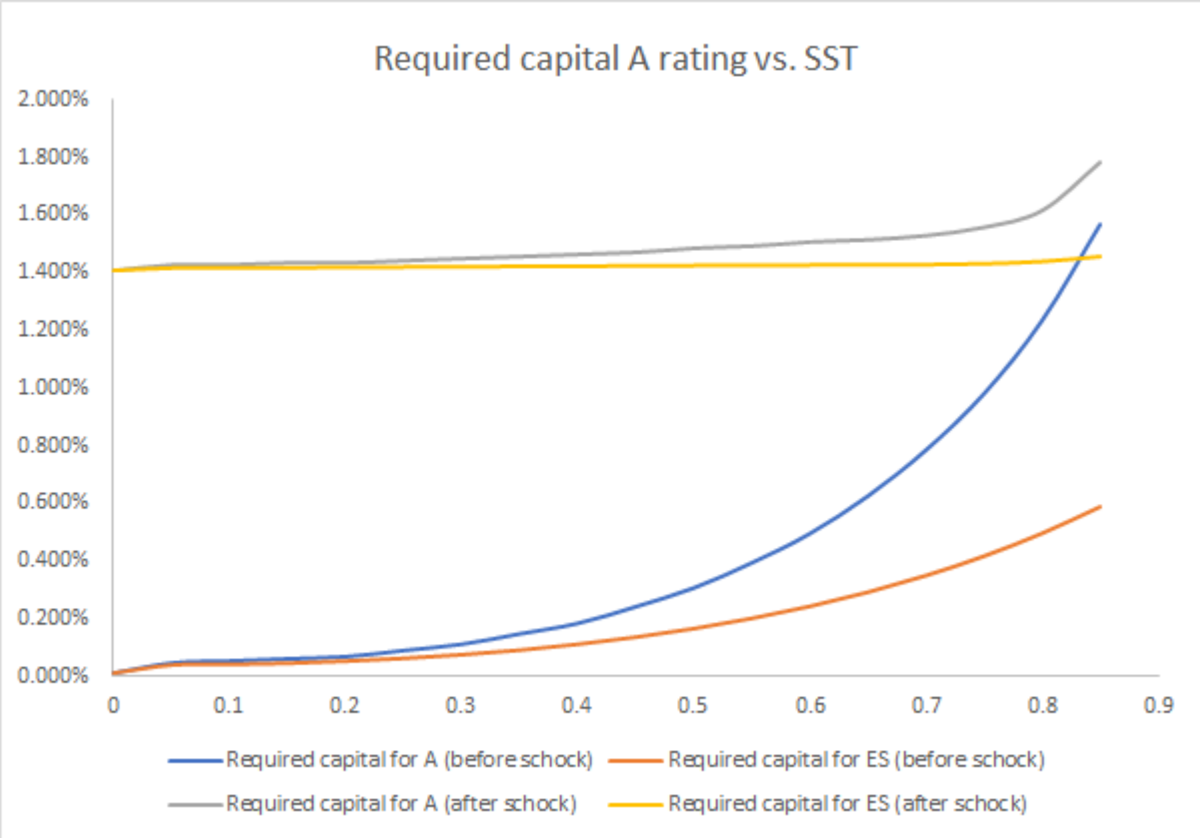
6 Results

Following table display the required capital for rating A and SST, both before and after shock

q	Required capital for A (before schock)	Required capital for SST (before schock)	Required capital for A (after schock)	Required capital for SST (after schock)
0	0.009%	0.009%	1.409%	**1.409%**
0.05	0.044%	0.037%	1.428%	1.418%
0.1	0.051%	0.039%	1.428%	1.418%
0.15	0.058%	0.043%	1.436%	1.419%
0.2	0.065%	0.050%	1.436%	1.420%
0.25	0.087%	0.060%	1.443%	1.421%
0.3	0.109%	0.073%	1.450%	1.422%
0.35	0.145%	0.089%	1.457%	1.423%
0.4	0.181%	0.110%	1.465%	1.424%
0.45	0.239%	0.134%	1.472%	1.425%
0.5	0.305%	0.164%	1.486%	1.426%
0.55	0.392%	0.200%	1.494%	1.428%
0.6	**0.493%**	**0.242%**	**1.508%**	**1.428%**
0.65	**0.624%**	**0.291%**	**1.515%**	**1.429%**
0.7	**0.783%**	**0.349%**	**1.530%**	**1.430%**
0.75	0.979%	0.417%	1.559%	1.433%
0.8	1.233%	0.496%	1.617%	1.440%
0.85	1.566%	0.589%	1.784%	1.458%
0.9	2.001%	0.696%	2.117%	1.495%
0.95	2.574%	0.812%	2.632%	1.572%
1	2.900%	0.870%	4.300%	**1.864%**

Table 4: Sensitivities with respect to q

As maximum required capital by both regimes we suggest required capital of 1.53% exposure which is on the safe side. The table shows that the assuming a shock of nineties has significant impact on the required capital. Following figure visualizes the sensitivities with respect to q.



Please note that this estimation can be reduced if we would choose less conservative assumption (especially LGD).

7 Appendix

7.1 Estimation of q

This section is devoted to estimation of q

Strictly speaking, for the modeling of systematic driver of default, we would need an economic scenario generator (ESG) explaining the systematic of the default event which might rather depend on the specific economic scenarios. However, this approach is elaborate and might exaggerated at this stage. Instead, we suggest a work around with which we can determine this parameter on the safe side.

To give an intuition about our approach, assume that at fixed time t in the past, we observed significantly higher defaults than assumed through the cycle default frequency (long run default frequency). Theoretically extreme event is partially triggered idiosyncratically and partially systematically. The question we would like to answer is: "With which mixture of systematic and idiosyncratic drivers we obtain maximal likelihood of the observed extreme event."

For this purpose, our ingredient consists of assumed through the cycle default probability PD (long run default probability), observed write-offs during real estate/mortgage shock in nineties with which can estimate the shock specific PD (denoted by BK) by means of observed top-down LGD in nineties. By using these inputs and assuming a large enough sample of mortgages (which is an evident assumption) we determine the likelihood for $0 \le q \le 1$ for fixed BK. Lucky, our likelihood function is strictly monotone increasing function of BK (see figure 4 for this statement) which ensure that we are rather on the safe side.

To focus on the ideas and keep the notation tractable the following steps are conducted as if the random variables were discrete. With a little abuse of notation, we will also use $\mathrm{X} = \mathrm{x}$ for the $\{w: X(w) = x\}$.

For the systematic driver S as introduced in (12) we have

$$\mathrm{E}\left[1_{\mathrm{S}+\mathrm{e_i} \le \mathrm{DT}} \, \mathrm{l} \, \mathrm{S} = \mathrm{s}\right]$$

$$= \mathrm{P}[\mathrm{e_i} \le \mathrm{DT} - \mathrm{S} \, \mathrm{l} \, \mathrm{S} = \mathrm{s}]$$

$$= \frac{\mathrm{P}[\mathrm{S} = \mathrm{s} \cap \mathrm{e_i} \le \mathrm{DT} - \mathrm{S}]}{\mathrm{P}[\mathrm{S} = \mathrm{s}]}$$

$$\mathrm{E}\left[1_{\mathrm{S}+\mathrm{e_i} \le \mathrm{DT}} \, \mathrm{l} \, \mathrm{S} = \mathrm{s}\right]$$

$$= \frac{\mathrm{P}[\mathrm{S} = \mathrm{s} \cap \mathrm{e_i} \le \mathrm{DT} - \mathrm{S}]}{\mathrm{P}[\mathrm{S} = \mathrm{s}]}$$

For fixed s, on the set $\{\mathrm{w}: \mathrm{S}(\mathrm{w}) = \mathrm{s}\}$ we have $\{\mathrm{w}: \mathrm{e_i} \le \mathrm{DT} - \mathrm{S}(\mathrm{w})\} = \{\mathrm{w}: \mathrm{e_i} \le \mathrm{DT} - \mathrm{s}\}$. Hence, the right hand side of the last equation can be replaced by $\frac{\mathrm{P}[\mathrm{S}=\mathrm{s} \cap \mathrm{e_i} \le \mathrm{DT}-\mathrm{s}]}{\mathrm{P}[\mathrm{S}=\mathrm{s}]}$ and due to independence of $\mathrm{e_i}$ and S it is equal to $\frac{\mathrm{P}[\mathrm{S}=\mathrm{s}] \, \mathrm{P}[\mathrm{e_i} \le \mathrm{DT}-\mathrm{s}]}{\mathrm{P}[\mathrm{S}=\mathrm{s}]}$.

Hence, we have

$$E\left[1_{S+e_i \le DT} \text{ l } S = s\right] = P\,[e_i \le DT - s],$$

which can be written as

$$E\left[1_{S+e_i \le DT} \text{ l } S = s\right] = F_{1-q^2}(DT - s),$$

where $F_{1-q^2}(.)$ Is the distribution function of e_i which is $N(0,1-q^2)$.

This implies

$$E\left[\lim \frac{1}{N} \sum_{n \le N} 1_{S+e_i \le DT} \mid S\right] = F_{1-q^2}(DT - S).$$

Now let s be fixed, the assumption , e_i $i = 1, 2, \ldots , N$ iid implies that $1_{s+e_i \le DT}$, $i = 1, 2, \ldots , N$ are also iid. Hence, by the law of large numbers, for fixed s and large N we have:

$$F_{1-q^2}(DT - s)$$

$$= E\left[1_{S+e_i \le DT} \text{ l } S = s\right]$$

$$= E\left[\, 1_{(s+e_i \le DT)}\right]$$

$$= \lim_{n \to \infty} \frac{1}{N} \sum_{i \le N} 1_{(s+e_i \le DT)}$$

Since this holds for all s, we can write

$$\lim_{N \to \infty} \frac{1}{N} \sum_{i \le N} 1_{(S+e_i \le DT)} = F_{1-q^2}(DT - S), \text{ P-almost surely.}$$

The result says that for large N, the portfolio distribution can be seen as distribution of function $F_{1-q^2}(DT - S)$ of systematic driver S which is also equal to $E\left[\lim \frac{1}{N} \sum_{n \le N} 1_{S+e_i \le DT} \mid S\right]$. The result is appealing and can be used to estimate the parameter q which will demonstrate next.

In nineties the mortgage sample N of Switzerland can be claimed to be large enough. However, we only know the write offs in % of exposure. With the help of observed real estate prices, we can estimate the top down LGD of mortgages (this LGD not same as the LGD of the insurance policies from point of insurer!) and obtain an estimate of top-down default frequency. But we still have an obstacle to overcome. Namely, realized systematic driver s is not known. We the following steps we introduce a work around to overcome this difficulty.

Let BK be estimated top-down default frequency in nineties.

We impose the following optimization problem:

$$\max_{0\le q\le 1}\left(P\left[F_{1-q^2}\ (DT-S)\in[BK,BK+\Delta BK]\right]\right)$$

Solution of above optimization gives the optimal q so that the expected default frequency in crises lies in a small neighbor of BK.

$$\max_{0\le q\le 1}\left(P\left[F_{1-q^2}\ (DT-S)\in[BK,BK+\Delta BK]\right]\right)$$

$$=\max_{0\le q\le 1}\left(P\left[-S\in F^{-1}_{1-q^2}\ ([BK,BK+\Delta BK])-DT\right]\right)$$

$$=\max_{0\le q\le 1}\left(P\left[S\in F^{-1}_{1-q^2}\ ([BK,BK+\Delta BK])-DT\right]\right)$$

$$=\max_{0\le q\le 1}\left(F_{q^2}\ (F^{-1}_{1-q^2}\ (BK+\Delta BK)-DT)-F_{q^2}\ (F^{-1}_{1-q^2}\ (BK)-DT)\right)$$

Second equation follows from fact that S and -S share the same distribution.

Diving of argument of max(.) in the last equation by ΔBK and taking $\Delta BK\to 0$ implies equivalent optimization problem:

$$\max_{0\le q\le 1}\left(\lim_{\Delta BK\to 0}\frac{F_{q^2}\ (F^{-1}_{1-q^2}\ (BK+\Delta BK)-DT)-F_{q^2}\ (F^{-1}_{1-q^2}\ (BK)-DT)}{\Delta BK}\right)$$

By chain rule this results in following equivalent optimization problem

$$\max_{0\le q\le 1}\left(\frac{f_{q^2}\left(F^{-1}_{1-q^2}(BK)-DT\right)}{f_{1-q^2}(F^{-1}_{1-q^2}(BK))}\right)=:\max_{0\le q\le 1}(H_{BK}(q))$$

Where $f_{q^2}(.)$ and $f_{1\_q^2}(.)$ are distribution density function of $N(0,q^2)$ resp. $N(0,1-q^2)$ random variable.

In nineties we observed 1.4% top-down annual write off. The observed value decay rented real estates (!) in whole crise period lies between 35% and 40% (which is higher than the observed value decay of own used real estates of less than 25%). On the very safe side (since duration in recovery clearly less than 6 or 7 years), we assume LGD of credit institution between 35% and 40% and again on the very safe side (without taking into account that our target group has certain rating quality) we obtain an estimated BK between less than 4% (which more than 10 times assumed top-down PD of our target group, see table 2 in in section 5).

Following graph show the how $H(q)$ behaves as a function of q.

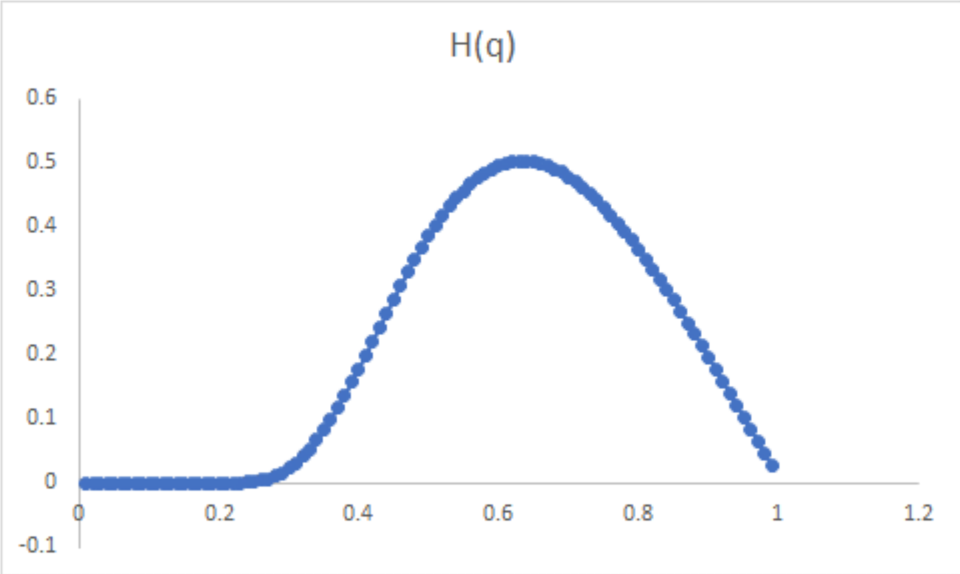


Figure 3: Likelihood of systematic parameter

In the above graph, the maximum of $H_{BK=4\%}$ (q) is attained at $\bar{q} = 63\%$.

Let us denote the $\bar{q}$(BK) solving the above optimization problem for $H_{BK}(q)$ which is a function of BK. The following graph shows that $\bar{q}$(BK) is strictly monotone increasing function of BK for BK larger than top-down PD = 03%.

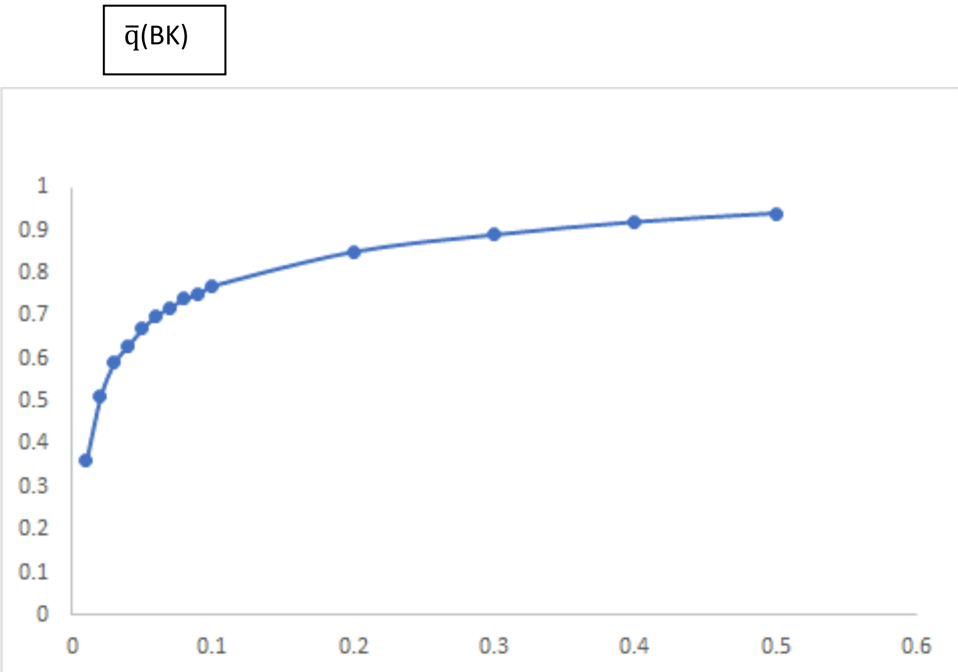


Figure 4: in y-axis is the optima $\bar{q}$ (BK) as function of BK (in x-axis).

From Figure 4 we first conclude that if

$$\text{top} - \text{down PD} < BK_1 < BK_2$$

then we have

$\bar{q}\,(BK_1) < \bar{q}(BK_2)$

Hence the required capital determined under the assumption BK_2 in crises will be large the required capital with BK_1. On the other hand the BK=0.04 (obtained by rounding up the estimation 0.038 which is the most severe real estate/mortgage shock since 1970) is determined from rented residential real estates which we also apply for own used real estates (own used objects are less likely to default than the rented objects). This means the actual BK should be lower than we are using. This proves that with our choice of systematic parameter $\bar{q}(0.04) = 0.63$ we are on the safe side.

Please also note that

$BK_1 < \ BK_2 < \mathrm{top} - \mathrm{down\ PD}$

then we have

$\bar{q}(BK_1) > \bar{q}(BK_2)$.

Hence, for $BK \in [0, \mathrm{top} - \mathrm{down\ PD}\ ]$ the function $\bar{q}$ is strictly monotone decreasing.

As a summary we conclude that the higher the deviation of the observed PD from the true PD, the higher systematic parameter q we can expect.